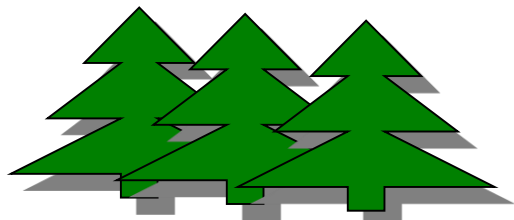中四選科之
高中課程資訊日
2023-2024
暨
中四選科講座

數學科(延伸部份)—單元二

嚴凱華老師
數學科科主任

高中的數學教育課程

高中數學課程
Senior Secondary Mathematics Curriculum

必修部分
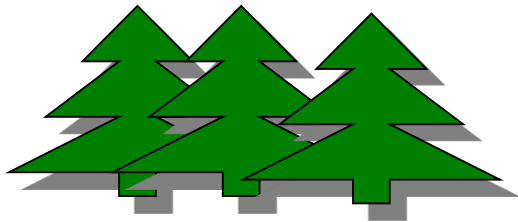**Compulsory Part**

延伸部分
Extended Part

單元一
（微積分與統計）
Module 1
(Calculus and Statistics)

單元二
（代數與微積分）
Module 2
(Algebra and Calculus)

延伸部分 M2

代數與微積分
(Algebra and Calculus)
重視深入的數學內容。

相關的必修部分課題： 指數及對數函數 續多項式 三角學
直線方程

基礎知識

- ✓ 二項式定理
- ✓ 數學歸納法
- ✓ 三角函數及圖像
- ✓ 複角的三角恆等式和公式
- ✓ 指數函數 (e^x)
- ✓ 對數函數 $(\ln x)$

微積分

- ✓ 極限
- ✓ 微分
- ✓ 不定積分
- ✓ 定積分
- ✓ 微積分的應用

代數

- ✓ 矩陣
- ✓ 三階方陣
- ✓ 線性方程
- ✓ 二維向量及三維向量

Binomial Theorem

We can see that all the coefficients in Pascal's Triangle can be rewritten in the form C_r^n.

The expansion of $(x + y)^n$ can be obtained by the following **binomial theorem**:

Binomial Theorem

If n is a positive integer, then

$$(x+y)^n = C_0^n x^n + C_1^n x^{n-1} y^1 + C_2^n x^{n-2} y^2 + \ldots + C_r^n x^{n-r} y^r + \ldots + C_n^n y^n$$

$$= \sum_{r=0}^{n} C_r^n x^{n-r} y^r$$

Using mathematical induction, we can prove the binomial theorem, that is,

$$(x+y)^n = C_0^n x^n + C_1^n x^{n-1} y^1 + C_2^n x^{n-2} y^2 + \ldots + C_r^n x^{n-r} y^r + \ldots + C_n^n y^n$$

for all positive integers n.

B. Binomial Theorem

Example

(a) Expand $(1-x)^5(5+mx)^3$ in ascending powers of x as far as the term in x^2.

(b) If the coefficient of x^2 in the expansion in (a) is 560, find the values of m.

Solution:

(a) $(1-x)^5(5+mx)^3$

$= [C_0^5(1)^5 + C_1^5(1)^4(-x) + C_2^5(1)^3(-x)^2 + \cdots]$

$\quad \times [C_0^3 5^3 + C_1^3 5^2(mx) + C_2^3 5(mx)^2 + \cdots]$

$= (1 - 5x + 10x^2 + \cdots)(125 + 75mx + 15m^2x^2 + \cdots)$

$= 125 + (75mx - 625x) + (15m^2x^2 - 375mx^2 + 1250x^2) + \cdots$

$= \underline{\underline{125 + (75m - 625)x + (15m^2 - 375m + 1250)x^2 + \cdots}}$

(b) $\because \quad$ Coefficient of $x^2 = 560$

$\therefore \quad 15m^2 - 375m + 1250 = 560$

$\quad\quad 15m^2 - 375m + 690 = 0$

$m^2 - 25m + 46 = 0$

$(m-2)(m-23) = 0$

$m = \underline{\underline{2 \text{ or } 23}}$

Compound Angle Formulas

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\tan (A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan (A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Double Angle Formulas

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\cos 2A = 2 \cos^2 A - 1$ or

$\cos 2A = 1 - 2 \sin^2 A$

$\cos^2 A = \dfrac{1}{2}(1 + \cos 2A), \quad \sin^2 A = \dfrac{1}{2}(1 - \cos 2A)$

Sum-to-product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Product-to-sum Formulas

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$$

Limits of Functions

A. Concept of Limit

Definition

Let a be a real number. We say that the limit of a function $f(x)$ is L when x approaches a if the value of $f(x)$ can be made arbitrarily close to L by taking values of x that are close to a but not equal to a. Symbolically, we write
$$\lim_{x \to a} f(x) = L .$$

Properties of Limits

Assume that $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, where L and M are real numbers and k is a real constant. Then

1. $\lim_{x \to a} k = k$ e.g. $\lim_{x \to 4} 2 = 2$

2. $\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x) = kL$ e.g. $\lim_{x \to 9} 3x = 3 \lim_{x \to 9} x = 27$

3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$

  e.g. $\lim_{x \to 4} (x - 2) = \lim_{x \to 4} x - \lim_{x \to 4} 2 = 4 - 2 = 2$

Properties of Limits

Assume that $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M$, where L and M are real numbers and k is a real constant. Then

4. $\lim\limits_{x \to a}[f(x)g(x)] = [\lim\limits_{x \to a} f(x)][\lim\limits_{x \to a} g(x)] = LM$

   e.g. $\lim\limits_{x \to 9} x^2 = \left(\lim\limits_{x \to 9} x\right)\left(\lim\limits_{x \to 9} x\right) = (9)(9) = 81$

5. $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = \dfrac{L}{M}$ e.g. $\lim\limits_{x \to 4} \dfrac{x}{x-2} = \dfrac{\lim\limits_{x \to 4} x}{\lim\limits_{x \to 4}(x-2)} = \dfrac{4}{2} = 2$

6. $\lim\limits_{x \to a} f(g(x)) = f(\lim\limits_{x \to a} g(x)) = f(M)$

   e.g. $\lim\limits_{x \to 4} \sqrt{x-2} = \sqrt{\lim\limits_{x \to 4} x - 2} = \sqrt{2}$

選修延伸單元時, 要特別注意:

* （1）數學科延伸單元並不是獨立科目。

* （2）在香港中學文憑公開考試，數學科延伸部份將設獨立考卷，考試成績亦將獨立顯示於中學文憑上。

考核模式

只考一試卷，時間為
兩小時三十分鐘。

試卷分為兩部，
全部題目均須作答。

甲部（佔50 分）
八題至九題短題目。

乙部（佔50 分）
四題至五題長題目。

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2017

MATHEMATICS   Extended Part
Module 2 (Algebra and Calculus)
Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

Candidate Number

11.　(a)　Using $\tan^{-1}\sqrt{2} - \tan^{-1}\left(\dfrac{\sqrt{2}}{2}\right) = \tan^{-1}\left(\dfrac{\sqrt{2}}{4}\right)$, evaluate $\displaystyle\int_0^1 \dfrac{1}{x^2+2x+3}\,dx$.　(3 marks)

(b)　(i)　Let $0 \le \theta \le \dfrac{\pi}{4}$. Prove that $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$ and $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$.

　　(ii)　Using the substitution $t = \tan\theta$, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{1}{\sin 2\theta + \cos 2\theta + 2}\,d\theta$.

(5 marks)

(c)　Prove that $\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{\sin 2\theta + 1}{\sin 2\theta + \cos 2\theta + 2}\,d\theta = \int_0^{\frac{\pi}{4}} \dfrac{\cos 2\theta + 1}{\sin 2\theta + \cos 2\theta + 2}\,d\theta$.　(2 marks)

(d)　Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{8\sin 2\theta + 9}{\sin 2\theta + \cos 2\theta + 2}\,d\theta$.　(3 marks)

表 3j : 2022 年文憑試數學必修部分及延伸部分的等級分佈
Table 3j : 2022 HKDSE Level distribution in Mathematics Compulsory Part and Extended Part

日校考生
Day School Candidates

延伸部分共設兩個選修單元，均採用同一評核標準。
若報考延伸部分，考生只可選擇兩個單元中的其中一個。
There are two optional modules in the Extended Part which are graded to the same standard.
Candidates are allowed to choose only one of the modules if they take the Extended Part.

日校考生出席數學必修部分及延伸部分的總人數
Total no. of day school candidates sitting both Compulsory Part and Extended Part: 6 067

數學必修部分成績 Attainment in Mathematics Compulsory Part	數學延伸部分成績 Attainment in Mathematics Extended Part								總數 Total
	5**	5*	5	4	3	2	1	U	
5**	142 (2.3%)	182 (3.0%)	124 (2.0%)	33 (0.5%)	3 (0.0%)			1 (0.0%)	485 (8.0%)
5*	67 (1.1%)	336 (5.5%)	512 (8.4%)	262 (4.3%)	67 (1.1%)	12 (0.2%)	2 (0.0%)		1 258 (20.7%)
5	7 (0.1%)	95 (1.6%)	455 (7.5%)	639 (10.5%)	342 (5.6%)	77 (1.3%)	14 (0.2%)	1 (0.0%)	1 630 (26.9%)
4	1 (0.0%)	11 (0.2%)	106 (1.7%)	458 (7.5%)	785 (12.9%)	582 (9.6%)	156 (2.6%)	29 (0.5%)	2 128 (35.1%)
3			1 (0.0%)	1 (0.0%)	43 (0.7%)	174 (2.9%)	170 (2.8%)	57 (0.9%)	446 (7.4%)
2						5 (0.1%)	33 (0.5%)	71 (1.2%)	109 (1.8%)
1								10 (0.2%)	10 (0.2%)
U								1 (0.0%)	1 (0.0%)
總數 Total	217 (3.6%)	624 (10.3%)	1 198 (19.7%)	1 393 (23.0%)	1 240 (20.4%)	850 (14.0%)	375 (6.2%)	170 (2.8%)	6 067 (100.0%)

科目 Subject		年份 Year	報考人數 No. entered	出席人數 No. sat	百分率 Percentage							
					5**	5*+	5+	4+	3+	2+	1+	U
數學 Mathematics	必修部分 Compulsory Part	2012	70 163	69 299	1.1	4.8	12.1	34.2	57.1	79.7	91.7	8.3
		2013	69 642	68 833	1.2	4.8	11.8	34.7	57.3	80.8	92.8	7.2
		2014	65 146	64 252	1.4	5.3	13.1	36.5	57.9	80.0	92.7	7.3
		2015	61 044	60 330	1.6	5.8	14.2	38.6	59.6	81.8	92.8	7.2
		2016	56 013	55 262	1.4	5.7	13.8	38.6	58.5	81.3	93.2	6.8
		2017	51 106	50 269	1.4	5.6	14.5	39.1	59.0	81.1	92.7	7.3
		2018	50 537	49 649	1.3	5.5	13.9	38.8	59.9	82.5	93.5	6.5
		2019	46 906	45 941	1.6	5.9	14.7	39.7	58.1	81.3	92.3	7.7
		2020	44 438	43 536	1.4	6.1	14.5	39.0	58.7	81.9	92.9	7.1
		2021	42 303	41 497	1.4	6.0	14.8	39.5	58.9	81.8	93.1	6.9
	延伸部分 (代數與微積分) Extended Part (Algebra and Calculus)	2012	8 227	8 091	3.0	12.0	31.4	56.1	79.8	92.5	98.0	2.0
		2013	6 974	6 734	3.1	12.9	33.2	58.8	81.7	93.0	98.3	1.7
		2014	5 699	5 528	3.5	15.5	36.4	62.8	82.8	93.6	98.3	1.7
		2015	5 280	5 094	3.7	17.2	41.2	63.1	82.1	93.3	98.4	1.6
		2016	4 606	4 462	3.5	14.3	37.3	61.9	81.7	93.1	98.4	1.6
		2017	4 316	4 165	4.2	16.4	43.1	61.5	82.4	93.2	98.5	1.5
		2018	4 556	4 401	3.5	14.5	37.4	59.7	81.0	92.8	98.0	2.0
		2019	4 345	4 231	3.6	15.2	38.3	59.9	80.2	92.8	98.2	1.8
		2020	4 348	4 209	3.9	15.2	37.2	59.4	81.7	92.8	98.3	1.7
		2021	4 063	3 936	4.1	16.4	39.7	60.4	81.6	93.0	98.3	1.7

選修延伸單元時有些甚麼地方要特別注意呢?

你願意再額外付出時間嗎?

	年度	必修部分		必修部分與一個單元
課時(小時)		250-313 10%-12.5%		$250 + \mathbf{125} = \mathbf{375}$ **15%**
中四 (每星期節數)	2024-2025	4A/B 7	4C/D 8	4A/B 7
中五 (每星期節數)	2025-2026	5A/B 7	5C/D 8	5A/B 7
中六 (每星期節數)	2026-2027	6	8	6A/B 6

選修延伸單元時有些甚麼地方要特別注意呢?

1. 個人能力、興趣

- ➤ 必須對數學有濃厚興趣
- ➤ 數學根底好
- ➤ 運算精密準確
- ➤ 願意與同學討論及研究
- ➤ 願意思考
- ➤ 願意花大量時計數及勇於克服數學難題。

選修延伸單元時有些甚麼地方要特別注意呢?

2. 個人目標/志向

- ➤ 升學
- ➤ 理想職業

學生修讀延伸單元對投考本地大學有何幫助?

個別院校都有學系將
單元一或單元二列為
附帶要求
或
優先考慮

https://www.edb.gov.hk/attachment/en/
curriculum-development/kla/ma/u.pdf
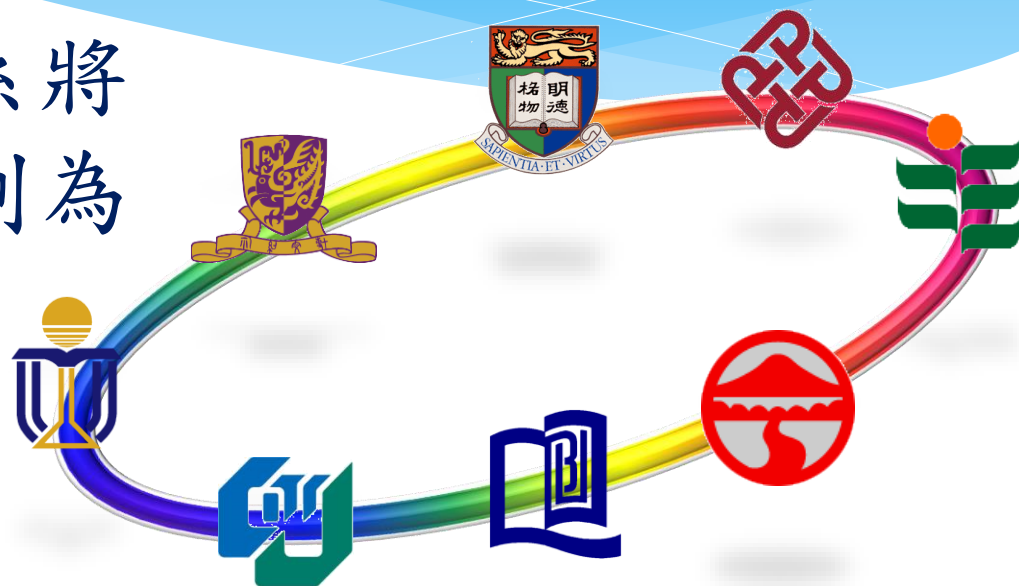
2024年大學聯招(JUPAS)中，八間資助大學均把單元一或單元二列為作其中一個選修科計算入學成績的數目。

General Entrance Requirements of UGC-funded Undergraduate Programmes under 2024 JUPAS

* An attainment at "Attained (A)" in "Citizenship and Social Development" is required for meeting the entrance requirement.

Universities	Basic Requirements (Chinese, English, Mathematics, Citizenship and Social Development* and Electives)	Counting M1/M2 as equivalent to an elective
City University of Hong Kong	332A33	✓
Hong Kong Baptist University	332A33	✓
Lingnan University	332A22	✓
The Chinese University of Hong Kong #	332A33	✓
The Education University of Hong Kong	332A22	✓
The Hong Kong Polytechnic University	332A33	✓
The Hong Kong University of Science and Technology #	332A33	✓
The University of Hong Kong	332A33	✓

The requirements are subject to relevant University committees' final approval.

Level 2

Comments

The candidate communicates and expresses relevant ideas using mathematical language and notations.

He/She employs <u>routine techniques</u> to solve the system of linear equations in Question 5. He/She is able to use differentiation to find the derivative in Question 9.

He/She is able to <u>perform straightforward</u> trigonometric proofs in Question 11(b)(i) and is also able to perform straightforward matrix manipulations in Question 12(b).

It can be concluded that the candidate demonstrates <u>basic knowledge</u> and understanding of the concepts underpinning algebra and calculus in the curriculum.

Level 5

Comments

The candidate demonstrates <u>comprehensive knowledge and understanding of the concepts</u> underpinning algebra and calculus in the curriculum <u>by applying them successfully at a</u> sophisticated level to <u>a wide range of unfamiliar situations</u> in Questions 9, 10, 11 and 12.

He/She is able to communicate and express views and arguments precisely and logically using mathematical language, notations and diagrams, such as in Questions 1, 2, 3, 4, 6, 8, 9, 11 and 12(b).

He/She also provides complex mathematical proofs in a logical, rigorous and concise manner in Questions 6(a), 11(c) and 12(a).

It can be concluded that the candidate <u>has the ability to integrate knowledge and skills from different areas of the curriculum</u> in handling complex tasks using a variety of strategies.

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Examinations and
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Category A: Senior Secondary Subjects

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- Assessment Information
- Examination Administration
- Important Dates
- FAQs on HKDSE
- Information Corner
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Category A - HKDSE Core Subjects: Mathematics - Samples of Candidates' Performance

Remark: Most of the samples of candidates' performance are in image format. If you have difficulty in reading the images, you may contact Hong Kong Blind Union for support services.

▶ **2020 Exam**	27/10/2020
▶ **2019 Exam**	25/10/2019
▶ **2018 Exam**	
▶ **2017 Exam**	
▶ **2016 Exam**	
▶ **2015 Exam**	
▶ **2014 Exam**	
▶ **2013 Exam**	
▶ **2012 Exam**	

有關考獲不同級別的
例子，可參考考評局
歷年考生表現示例

Sample Performance

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Category A - HKDSE Core Subjects: Mathematics - Samples of Candidate

Performance - 2020 Exam - Extended Part - Module 2

- ▶ Introduction
- ▶ Level 5
- ▶ Level 5 additional sample
- ▶ Level 4
- ▶ Level 4 additional sample
- ▶ Level 3
- ▶ Level 3 additional sample
- ▶ Level 2
- ▶ Level 2 additional sample
- ▶ Level 1

完